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Generalized Maxwell Equations and Quantum Mechanics. I. Dirac Equation for the Free Electron

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Some preliminary results presented in two previous papers are expanded upon. In the first it was shown that the Maxwell equations are equivalent to a nonlinear Dirac-like spinor equation. In the present paper it is shown that, in that formalism, the Dirac equation for the free electron is susceptible to a puzzling reinterpretation. In fact, it is shown that the Dirac equation is equivalent to the Maxwell equations for an electromagnetic field generated by two currents: one electric in nature and one, magnetic-monopolar. The elaboration of this result brings a nonlinear generalization of Maxwell's equations, as well as a nonlinear Dirac-like equation fully equivalent to them, from which both the electron mass as well as the magnetic monopole mass appear to be fully electromagnetic in nature, and the magnetic monopole to be tachyonic. The corresponding nonlinear Dirac equation reduces, under suitable approximations, to the ordinary Dirac equation for the free electron.

1. INTRODUCTION

Some time ago the author presented two papers (Campolattaro 1980*a*,*b*) (hereafter referred to as I and II) as the output of a research program which, due to severe conditions, had to be postponed. The relief of those circumstances has allowed the pursuit of this research and some results are presented here.

In I and II it was shown that for any electromagnetic field tensor $F^{\mu\nu}$ it is possible to find a spinor Ψ such that one has

$$F^{\mu\nu}=\bar{\Psi}S^{\mu\nu}\Psi \tag{1}$$

where $\bar{\Psi}$ is the Dirac conjugate of the spinor Ψ and $S^{\mu\nu}$ is the spin operator defined by

$$S^{\mu\nu}=\frac{i}{2}\gamma^{[\mu}\gamma^{\nu]} \tag{2}$$

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where

$$\gamma^{[\mu}\gamma^{\nu]} = \tfrac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu) \tag{3}$$

and the γ's are the Dirac matrices satisfying the anticommutation condition

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu} \tag{4}$$

with $\eta^{\mu\nu}$ the Minkowski metric tensor given by

$$\eta^{\mu\nu} = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array} \tag{5}$$

and the Dirac matrices adopted are those of the Dirac representation, with

$$\gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3 \tag{6}$$

and

$$\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0 \tag{7}$$

In this representation the dual tensor (the Einstein sum convention is adopted throughout)

$$*F^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\sigma\tau}F_{\sigma\tau} \tag{8}$$

where $\varepsilon^{\mu\nu\sigma\tau}$ is the Ricci pseudotensor with entry $+1$ if the parity of the permutation $\mu\nu\sigma\tau$ of the indices 0, 1, 2, 3 is even, and -1 if odd, and entry zero if two or more indices are equal, assumes the form

$$*F^{\mu\nu} = \bar{\Psi}\gamma^5 S^{\mu\nu}\Psi \tag{9}$$

and the Maxwell equations read (a comma followed by an index represents the partial derivative with respect to the variable with that index)

$$(\bar{\Psi}S^{\mu\nu}\Psi)_{,\mu} = j^\nu \tag{10}$$

and

$$(\bar{\Psi}\gamma^5 S^{\mu\nu}\Psi)_{,\mu} = 0 \tag{11}$$

The electromagnetic field tensor involves then the spinor operator. Moreover, the duality rotation (Rainich, 1925; Misner and Wheeler, 1957) by the complexion α, namely

$$\bar{F}^{\mu\nu} = F^{\mu\nu}\cos\alpha + *F^{\mu\nu}\sin\alpha \tag{12}$$

is equivalent to a Touschek-Nishijima (Touschek, 1957; Nishijima, 1957) transformation for the spinor Ψ to the spinor Ψ' given by

$$\Psi' = e^{\gamma^5 \alpha/2}\Psi \tag{13}$$

with

$$e^{\gamma^5 \alpha} = \cos\alpha + \gamma^5 \sin\alpha \tag{14}$$

and

$$\cos\alpha = \frac{\bar{\Psi}\Psi}{\rho} \tag{15}$$

$$\sin\alpha = \frac{\bar{\Psi}\gamma^5\Psi}{\rho} \tag{16}$$

ρ being the positive square root of

$$\rho^2 = (\bar{\Psi}\Psi)^2 + (\bar{\Psi}\gamma^5\Psi)^2 \tag{17}$$

and the Touschek-Nishijima transformation is the simplest of the chiral transformations (Coleman and Glashow, 1962).

In this spinor representation a vector and a pseudovector appear quite naturally, namely the vector $\bar{\Psi}\gamma^\mu\Psi$ and the pseudovector $\bar{\Psi}\gamma^5\gamma^\mu\Psi$, which are orthogonal, i.e.,

$$(\bar{\Psi}\gamma_\mu\Psi)(\bar{\Psi}\gamma^5\gamma^\mu\Psi) = 0 \tag{18}$$

and with the same moduli ρ, i.e.,

$$(\bar{\Psi}\gamma^\mu\Psi)(\bar{\Psi}_{\gamma\mu}\Psi) = \rho^2 \tag{19}$$

$$(\bar{\Psi}\gamma^5\gamma^\mu\Psi)(\bar{\Psi}\gamma^5\gamma^\mu\Psi) = \rho^2 \tag{20}$$

which can then be normalized by

$$\xi^\mu = \frac{1}{\rho}(\bar{\Psi}\gamma^\mu\Psi) \tag{21}$$

$$\eta^\mu = \frac{1}{\rho}(\bar{\Psi}\gamma^5\gamma^\mu\Psi) \tag{22}$$

with the properties

$$\xi^\mu\eta_\mu = \eta^\mu\xi_\mu = 0 \tag{23}$$

$$\xi^\mu\xi_\mu = \eta^\mu\eta_\mu = 1 \tag{24}$$

Moreover, the two spinor Maxwell equations (10) and (11) were shown to be equivalent to a single nonlinear first-order equation for the spinor Ψ, namely

$$\gamma^{\mu}\Psi_{,\mu} = -i\gamma^{\mu}\frac{e^{\gamma^5\alpha}}{\rho}\{I_m(\bar{\Psi}_{,\mu}\Psi) - j_{\mu} - \gamma^5 I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi)\}\Psi \qquad (25)$$

These results, namely the connection between the electromagnetic field tensor and the spin operator, as well as that of the duality rotation with a chirality transformation and the similarity of the spinor equivalent equation of the Maxwell equations with the classical Dirac equation for the electron, have titillated my imagination to the point of questioning if the relationship between the classical electromagnetic theory and relativistic quantum mechanics was not merely formal but more profound.

2. THE GENERALIZED MAXWELL EQUATIONS

As Maxwell himself did, I have ignored the possibility of existence of magnetic monopoles. However, the existence of monopoles makes the electromagnetic field theory so symmetric then over and over one likes to think about the reality of magnetic monopoles. Let us therefore assume that together with an electric current j_{μ}, there exists also a magnetic monopole current g_{μ}. In this spinor formalism the Maxwell equations (10) and (11) read

$$(\bar{\Psi}S^{\mu\nu}\Psi)_{,\mu} = j^{\nu} \qquad \text{[equation (10)]}$$

and

$$(\bar{\Psi}\gamma^5 S^{\mu\nu}\Psi)_{,\mu} = g^{\nu} \qquad (26)$$

and it is easily shown that the spinor equation (25), in the presence of magnetic monopoles, reads

$$\gamma^{\mu}\Psi_{,\mu} = -i\gamma^{\mu}\frac{e^{\gamma^5\alpha}}{\rho}\{I_m(\bar{\Psi}_{,\mu}\Psi) - j_{\mu} - \gamma^5[I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi) - g_{\mu}]\}\Psi \qquad (27)$$

3. TWO GAUGE CONDITIONS

At this point let me point out that in the presented spinor representation of the Maxwell equations, the six real components of the electric and magnetic fields have been replaced by a spinor Ψ, which exhibits eight real functions in its components, so that two of these parameters are not needed. One is therefore free to impose two conditions on the spinor Ψ so as to reduce the necessary parameters to the six required for the description of the electromagnetic field. Let us choose the two gauge conditions:

$$I_m[\bar{\Psi}\Box\Psi]=0 \tag{28}$$

$$I_m[\bar{\Psi}\gamma^5\Box\Psi]=0 \tag{29}$$

where $\Box$ represents the ordinary D'Alembertian.

The reason for this choice resides in the fact that, as is readily seen, the vector $I_m(\bar{\Psi}_{,\mu}\Psi)$ and the pseudovector $I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi)$ become solenoidal, i.e.,

$$\{\eta^{\mu\nu}I_m(\bar{\Psi}_{,\nu}\Psi)\}_{,\mu}=0 \tag{30}$$

and

$$\{\eta^{\mu\nu}I_m(\bar{\Psi}_{,\nu}\gamma^5\Psi)\}_{,\mu}=0 \tag{31}$$

where $\eta^{\mu\nu}$ is the Minkowski metric tensor given by (5). This property will be used in the next section.

4. THE PHYSICAL INTERPRETATION OF THE VECTORS $I_m(\Psi_{,\mu}\Psi)$ AND $I_m(\Psi_{,\mu}\gamma^5\Psi)$

For the physical interpretation of these two vectors, consider equation (27), in which these two vectors are neglected, i.e.,

$$\gamma^\mu\Psi^0_{,\mu}-i\gamma^\mu\frac{e^{\gamma^5\alpha^0}}{\rho^0}(j_\mu-\gamma^5 g_\mu)\Psi^0=0 \tag{32}$$

Equation (32) can be also written identically as follows:

$$\begin{aligned}\gamma^\mu\Psi^0_{,\mu}=-i\gamma^\mu\frac{e^{\gamma^5\alpha^0}}{\rho^0}\{&I_m(\bar{\Psi}^0_{,\mu}\Psi^0)-[I_m(\bar{\Psi}^0_{,\mu}\Psi^0)+j_\mu]\\&-\gamma^5[I_m(\bar{\Psi}^0_{,\mu}\gamma^5\Psi^0)-(I_m(\bar{\Psi}^0_{,\mu}\gamma^5\Psi^0)+g_\mu)]\}\Psi^0\end{aligned} \tag{33}$$

Because of the two gauge conditions (28) and (29), the vectors $I_m(\bar{\Psi}^0_{,\mu}\Psi^0)$ and $I_m(\bar{\Psi}^0_{,\mu}\gamma^5\Psi^0)$ can be considered as two conserved currents and thence, from equation (27) and its demonstrated equivalence to Maxwell's equations, they define an electromagnetic field satisfying the equations

$$(\bar{\Psi}^0 S^{\mu\nu}\Psi^0)_{,\nu}=\eta^{\mu\nu}I_m(\bar{\Psi}^0_{,\nu}\Psi^0)+j^\mu \tag{34}$$

and

$$(\bar{\Psi}^0\gamma^5 S^{\mu\nu}\Psi^0)_{,\nu}=\eta^{\mu\nu}I_m(\bar{\Psi}^0_{,\nu}\gamma^5\Psi^0)+g^\mu \tag{35}$$

On the other hand, since $I_m(\bar{\Psi}^0_{,\mu}\Psi^0)$ and $I_m(\bar{\Psi}^0_{,\mu}\gamma^5\Psi^0)$ are conserved, we can use the above-mentioned results, which ensure the existence of a spinor Φ such that

$$(\bar{\Phi}S^{\mu\nu}\Phi)_{,\nu} = -\eta^{\mu\nu}I_m(\bar{\Psi}^0_{,\nu}\Psi^0) = \eta^{\mu\nu}I_m(\bar{\Psi}^0\Psi^0_{,\nu}) \tag{36}$$

and

$$(\bar{\Phi}\gamma^5S^{\mu\nu}\Phi)_{,\nu} = -\eta^{\mu\nu}I_m(\bar{\Psi}^0_{,\nu}\gamma^5\Psi^0) = \eta^{\mu\nu}I_m(\bar{\Psi}^0\gamma^5\Psi^0_{,\nu}) \tag{37}$$

so that equations (34) and (35) read

$$\{\bar{\Psi}^0S^{\mu\nu}\Psi^0 + \bar{\Phi}S^{\mu\nu}\Phi\}_{,\nu} = j^\mu \tag{38}$$

and

$$\{\bar{\Psi}^0\gamma^5S^{\mu\nu}\Psi^0 + \bar{\Phi}\gamma^5S^{\mu\nu}\Phi\}_{,\nu} = g^\mu \tag{39}$$

The tensor

$$\bar{\Psi}^0S^{\mu\nu}\Psi^0 + \bar{\Phi}S^{\mu\nu}\Phi = F^{\mu\nu} \tag{40}$$

is therefore the Maxwell electromagnetic field associated with the currents j^μ and g^μ. By putting for the sake of simplicity

$$\bar{\Psi}^0S^{\mu\nu}\Psi^0 = f^{\mu\nu} \tag{41}$$

$$\bar{\Phi}S^{\mu\nu}\Phi = P^{\mu\nu} \tag{42}$$

equation (40) gives

$$f^{\mu\nu} = F^{\mu\nu} - P^{\mu\nu} \tag{43}$$

Thence the neglect in equation (27) of the vectors $I_m(\bar{\Psi}_{,\mu}\Psi)$ and $I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi)$ is equivalent to studying not the physical field $F^{\mu\nu}$, but the field $f^{\mu\nu}$. This tensor $P^{\mu\nu}$, defined by equations (36) and (37), is called the "vacuum polarization tensor." Here, however, as well as in later sections, the term "vacuum polarization" is used loosely for analogic reasons and for sake of economy of terminology. Whether it has something to do with what is ordinarily understood by vacuum polarization, say, e.g., in relationship with the work of Heisenberg and Euler (1936), is a question whose answer is left to further investigations. Thus, considering equation (27) without the terms $I_m(\bar{\Psi}_{,\mu}\Psi)$ and $I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi)$, i.e., equation (32), is equivalent, in other terms, to neglecting the vacuum polarization effect. I call the tensor field $f^{\mu\nu}$ the "bare field."

5. A REINTERPRETATION OF THE DIRAC EQUATION FOR THE FREE ELECTRON

Let us consider the Dirac equation for the free electron, i.e.,

$$(\gamma^\mu\partial_\mu + im)\Psi = 0 \tag{44}$$

By multiplying equation (44) on the left by $\bar{\Psi}\gamma^{\nu}$ one has

$$\bar{\Psi}\gamma^{\nu}\gamma^{\mu}\partial_{\mu}\Psi + im\bar{\Psi}\gamma^{\nu}\Psi = 0 \tag{45}$$

From equations (2) and (3), equation (45) reads

$$2i\bar{\Psi}S^{\mu\nu}\Psi_{,\mu} + \eta^{\mu\nu}\bar{\Psi}\partial_{\mu}\Psi + im\bar{\Psi}\gamma^{\nu}\Psi = 0 \tag{46}$$

By taking the Hermitian conjugate of equation (46), one has

$$2i\bar{\Psi}_{,\mu}S^{\mu\nu}\Psi - \eta^{\mu\nu}(\partial_{\mu}\bar{\Psi})\Psi + im\bar{\Psi}\gamma^{\nu}\Psi = 0 \tag{47}$$

and by adding equations (46) and (47), one obtains, for the antisymmetry of $S^{\mu\nu}$,

$$(\bar{\Psi}S^{\nu\mu}\Psi)_{,\mu} = \eta^{\mu\nu}Im(\bar{\Psi}\Psi_{,\mu}) + m\bar{\Psi}\gamma^{\nu}\Psi \tag{48}$$

Similarly, by multiplying equation (44) on the left by $\bar{\Psi}\gamma^{5}\gamma^{\nu}$ and by repeating the steps followed in the previous lines, one has

$$(\bar{\Psi}\gamma^{5}S^{\nu\mu}\Psi)_{,\mu} = \eta^{\mu\nu}Im(\bar{\Psi}\gamma^{5}\Psi_{,\mu}) \tag{49}$$

Equations (48) and (49) are completely equivalent to the Dirac equation (44). Therefore, one has, by using the results expressed by equations (10) and (26), that the Dirac equation is equivalent to the Maxwell equations for an electromagnetic field $\bar{F}^{\mu\nu}$ defined by

$$\bar{F}^{\mu\nu} = \bar{\Psi}S^{\mu\nu}\Psi \tag{50}$$

and thence

$${}^{*}\bar{F}^{\mu\nu} = \bar{\Psi}\gamma^{5}S^{\mu\nu}\Psi \tag{51}$$

generated by the two currents

$$j^{\mu} = \eta^{\mu\nu}Im(\bar{\Psi}\Psi_{,\nu}) + m(\bar{\Psi}\gamma^{\mu}\Psi) \tag{52}$$

and

$$g^{\mu} = \eta^{\mu\nu}Im(\bar{\Psi}\gamma^{5}\Psi_{,\nu}) \tag{53}$$

the first electronic in nature and the second magnetic monopolar, or simply monopolar. The two gauge conditions (28) and (29) are automatically satisfied because each of the four components of the Dirac spinor satisfies the Klein–Gordon equation and the current $m\bar{\Psi}\gamma^{\mu}\Psi$ is conserved.

I elaborate this result below.

6. FURTHER GENERALIZED MAXWELL EQUATIONS

From here on I will study further developments in the bare field approximation, and omit the superscript zero for the sake of simplicity and without fear of ambiguity.

As pointed out in the introduction, in the spinor representation of electromagnetism, a vector $\Psi\gamma^\mu\Psi$ and a pseudovector $\bar{\Psi}\gamma^5\gamma^\mu\Psi$ come out quite naturally; their algebraic properties with respect to the electromagnetic field tensor, its dual, and the energy-momentum tensor were shown in II. But are they only mathematical entities, deprived of any physical meaning?

In I, it was shown that the spinor Ψ cannot be a neutrettor, in the sense of Corson (1955); thence these two vectors carry a charge and so these vectors are charge currents. However, in general one does not have that these two new currents are conserved, so I introduce two real scalars m and n and in correspondence a vector m_μ and a pseudovector n_μ, both real, defined by

$$m_\mu = m(\bar{\Psi}\gamma_\mu\Psi) \tag{54}$$

and

$$n_\mu = -in(\bar{\Psi}\gamma^5\gamma_\mu\Psi) \tag{55}$$

so that both m_μ and n_μ are solenoidal, i.e.,

$$m^\mu_{,\mu} = 0 \tag{56}$$

$$n^\mu_{,\mu} = 0 \tag{57}$$

What happens to these currents?

Given an electrical current density j_μ and a monopole current density g_μ, they generate through the field equations (10) and (26) some other currents $m\bar{\Psi}\gamma^\mu\Psi$ and $-in\bar{\Psi}\gamma^5\gamma^\mu\Psi$ which are then fed back in the field, which in turn produces another electromagnetic field and another spinor and thence two more currents $m\bar{\Psi}\gamma^\mu\Psi$ and $-in\bar{\Psi}\gamma^5\gamma^\mu\Psi$ and so on *ad infinitum*; at the limit this feedback process gives another generalized spinor Maxwell equation which reads

$$\gamma^\mu\Psi_{,\mu} = i\gamma^\mu\frac{e^{\gamma^5\alpha}}{\rho}\{j_\mu + m_\mu - \gamma^5(g_\mu + n_\mu)\}\Psi \tag{58}$$

for the new electromagnetic field after the feedback of the two conserved currents (54) and (55); equation (58) can be rewritten in the form

$$\gamma^\mu\Psi_{,\mu} = i\gamma^\mu\frac{e^{\gamma^5\alpha}}{\rho}(j_\mu - \gamma^5 g_\mu)\Psi + i\gamma^\mu\frac{e^{\gamma^5\alpha}}{\rho}(m_\mu - \gamma^5 n_\mu)\Psi \tag{59}$$

The second term on the right side can be rewritten, after (54), (55), (21), and (22), as follows:

$$i\gamma^\mu\frac{e^{\gamma^5\alpha}}{\rho}(m_\mu - \gamma^5 n_\mu)\Psi = im\, e^{-\gamma^5\alpha}\gamma^\mu\xi_\mu\Psi + n\, e^{-\gamma^5\alpha}\gamma^5\gamma^\mu\eta_\mu\Psi \tag{60}$$

and taking account of (A6) and (A8) of Appendix A, equation (60) reads

$$i\gamma^{\mu}\frac{e^{\gamma^5\alpha}}{\rho}(m_{\mu}-\gamma^5 n_{\mu})=(im+n)\Psi \tag{61}$$

so that the generalized spinor Maxwell equation (59) reads

$$\gamma^{\mu}\Psi_{,\mu}=i\gamma^{\mu}\frac{e^{\gamma^5\alpha}}{\rho}(j_{\mu}-\gamma^5 g_{\mu})+i(m-in)\Psi \tag{62}$$

or

$$\gamma^{\mu}\left\{\partial_{\mu}-i\frac{e^{\gamma^5\alpha}}{\rho}(j_{\mu}-\gamma^5 g_{\mu})\right\}\Psi-i(m-in)\Psi=0 \tag{63}$$

7. MAGNETIC MONOPOLES AS TACHYONS

The mass term in equation (63), namely $m-in$, is a complex number whose real term has to be identified with the rest mass of the electron, while the imaginary part has to be identified with the rest mass of the magnetic monopole. This result indicates that the magnetic monopoles are tachyons or superluminal particles. Historically, the idea of superluminal particles precedes the theory of relativity (Thomson, 1889; Heaviside, 1892; Des Coudres, 1990; Sommerfeld, 1904), as pointed out by Recami and Mignani (1974*b*). In the postrelativity era, Recami and Mignani (1974*a*) proposed the magnetic monopoles as superluminal objects. The same authors extended the theory of relativity to handle both bradyons as well as tachyons [see Mignani and Recami (1974*b*) its extensive bibliography], eliminating all the paradoxes associated with superluminal objects, legitimizing their consideration in nature. In subsequent papers (Mignani and Recami, 1974*a*, 1975; Recami and Mignani, 1974*b*,*c*, 1976; Corben and Honig, 1975; Corben, 1975; Recami, 1976, 1987) magnetic monopoles as tachyons were further investigated. In the generalized Maxwell equations, both the mass of the electron as well as that of the magnetic monopole appear to be fully electromagnetic in nature.

8. THE MASS EQUATIONS

The masses m and n which appear in equation (40) cannot be arbitrary since they have to be such that the conservation conditions (56) and (57) are satisfied.

Due to (54) and (55), these conservation conditions read

$$m_{,\mu}(\bar{\Psi}\gamma^{\mu}\Psi)+m(\bar{\Psi}\gamma^{\mu}\Psi)_{,\mu}=0 \tag{64}$$

and

$$n_{,\mu}(\bar{\Psi}\gamma^5\gamma^\mu\Psi)+n(\bar{\Psi}\gamma^5\gamma^\mu\Psi)_{,\mu}=0 \tag{65}$$

On the other hand, equations (B10) and (B11) of Appendix B hold, together with (B9), and thence equations (64) and (65) read

$$m_{,\mu}(\bar{\Psi}\gamma^\mu\Psi)=2i\rho m A_\mu\eta^\mu-2mn\rho\cos\alpha \tag{66}$$

and

$$n_{,\mu}(\bar{\Psi}\gamma^5\gamma^\mu\Psi)=-2i\rho n A_\mu\xi^\mu-2imn\rho\sin\alpha \tag{67}$$

with A_μ given by (B9), i.e.,

$$A_\mu=\frac{1}{\rho}(j_\mu\sin\alpha-g_\mu\cos\alpha) \tag{68}$$

or, equivalently, ρ being $\neq 0$,

$$[\ln(m)]_{,\mu}\xi^\mu+2n\cos\alpha=2iA_\mu\eta^\mu \tag{69}$$

and

$$[\ln(n)]_{,\mu}\eta^\mu+2im\sin\alpha=-2iA_\mu\xi^\mu \tag{70}$$

From equation (69) one has, in the hypothesis

$$\cos\alpha\neq 0 \tag{71}$$

$$n=\frac{2iA_\mu\eta^\mu-[\ln(m)]_{,\mu}\xi^\mu}{2\cos\alpha} \tag{72}$$

i.e., once m is known, n is known algebraically.

From equations (59) and (57) one has an equation involving only m, i.e.,

$$\left\{\ln\left[\frac{2iA_\mu\eta^\mu-[\ln(m)]_{,\mu}\xi^\mu}{2\cos\alpha}\right]\right\}_{,\nu}n^\nu+2im\sin\alpha=-2iA_\mu\xi^\mu \tag{73}$$

9. THE MASS FORMULAS

Fortunately, one does not have to struggle with the mathematical intricacies of the mass equations of the previous section in order to determine the functional dependence of the masses on the spinor Ψ. In fact, by replacing the terms $m_{,\mu}(\Psi\gamma^\mu\Psi)$ and $n_{,\mu}(\Psi\gamma^5\gamma^\mu\Psi)$, respectively, with $-m(\Psi\gamma^\mu\Psi)_{,\mu}$ and $-n(\Psi\gamma^5\gamma^\mu\Psi)_{,\mu}$ from equations (65) and (66), one readily has

$$m=\frac{i}{2}\frac{2iA_\mu(\bar{\Psi}\gamma^\mu\Psi)-(\bar{\Psi}\gamma^5\gamma^\mu\Psi)_{,\mu}}{\bar{\Psi}\gamma^5\Psi} \tag{74}$$

and

$$n=\frac{1}{2}\frac{2iA_{\mu}(\bar{\Psi}\gamma^{5}\gamma^{\mu}\Psi)+(\bar{\Psi}\gamma^{\mu}\Psi)_{,\mu}}{\bar{\Psi}\Psi} \tag{75}$$

These are just the values of m and n as evaluated in Appendix. B. Therefore, the values of m and n given by equations (74) and (75) ensure the self-consistency of equation (41) and the continuity conditions (57) and (58) are automatically satisfied.

10. DIRAC EQUATION FOR THE FREE ELECTRON

In the case of nonexistence of magnetic monopoles, then both g_{μ} and n vanish and equation (64) gives

$$\left\{\gamma^{\mu}\partial_{\mu}-i\frac{e^{\gamma^{5}\alpha}}{\rho}j_{\mu}\right\}\Psi-im\Psi=0 \tag{76}$$

In the absence of free electrical charges, equation (76) reduces to

$$\gamma_{\mu}\partial_{\mu}\Psi-im\Psi=0 \tag{77}$$

In this case, however, equation (B7) gives

$$(\bar{\Psi}\gamma^{\mu}\Psi)_{,\mu}=0 \tag{78}$$

so that the conservation condition (57) is satisfied with m constant and equation (77) is the Dirac equation for the free electron.

However, if magnetic monopoles do exist, in the absence of free charges, i.e., $j_{\mu}=0$ and $g_{\mu}=0$, m does not need to be constant; in fact, equations (74) and (75) reduce to

$$m=-\frac{i}{2}\frac{(\bar{\Psi}\gamma^{5}\gamma^{\mu}\Psi)_{,\mu}}{\bar{\Psi}\gamma^{5}\Psi} \tag{79}$$

and

$$n=\frac{1}{2}\frac{(\bar{\Psi}\gamma^{\mu}\Psi)_{,\mu}}{\bar{\Psi}\Psi} \tag{80}$$

so that equation (63) gives

$$\gamma^{\mu}\partial_{\mu}\Psi-i(m-in)\Psi=0 \tag{81}$$

which is a Dirac equation, nonlinear, due to the functional dependence on Ψ of both m and n through equations (79) and (80). Nonlinear Dirac equations where indeed the nonlinearity is confined to the mass term have recently been studied (Kersten, 1983; Strauss and Vázquez, 1986; Furlan and Raczka, 1986) and some solutions have been found for particular cases (Rãgnada and Usón, 1980; Fushchich and Shtelen, 1983; Steeb and Devel, 1984).

11. CONCLUSIONS

In this paper it has been shown that some preliminary results (Campolattaro, 1980*a,b*) can be extended to the point of revealing an intimate connection between the celebrated Maxwell equations of classical electromagnetism and relativistic quantum mechanics.

A suitable choice of gauge has allowed us to interpret the two terms which appear in the spinor equation equivalent to Maxwell's equations, namely $I_m(\bar{\Psi}_{,\mu}\Psi)$ and $I_m(\bar{\Psi}_{,\mu}\gamma^5\Psi)$, as coming from vacuum polarization effects. In this paper these vacuum polarization effects have been neglected and this approximation has been called the "bare field approximation." In this approximation, by a feedback process the Maxwell equations, in their spinor representation, have been generalized. The selected choice of gauge has allowed us to write a generalized spinor Maxwell equation. With this generalization the mass of the electron as well as that of the magnetic monopole appears to be fully electromagnetic in nature. The appearance, however, of a complex mass term reveals that while electrons are bradyons ($v<c$), the magnetic monopoles are tachyons ($v>c$), and this is in agreement with an idea of Recami and Mignani which dates back to 1974. The further pursuit of this subject is left to later investigations. The masses, however, are not constant, but are functionally dependent on the Maxwell spinor. Two mass equations have been written, and the mass functional dependence on the spinor Ψ has been deduced. The ordinary Dirac equation for the free electron has been derived under the simplified conditions of the nonexistence of magnetic monopoles and the absence of electric charges. However, if the existence of magnetic monopoles is not excluded, the free solution, i.e., the one with both j_μ and g_μ vanishing, requires both an electronic mass as well as a magnetic monopole mass, the latter not zero and both not constant, but functionally dependent on the Maxwell spinor Ψ. This result brings us to the most recent studies of nonlinear Dirac equations where the nonlinearity resides, in fact, in the mass term.

APPENDIX A

In I, it was shown that given three arbitrary spinors χ, Φ, and Ψ, one has

$$(\bar{\chi}\gamma^\mu\Psi)(\bar{\Phi}\gamma_\mu\Psi)=(\bar{\chi}\Psi)(\bar{\Phi}\Psi)+(\bar{\chi}\gamma^5\Psi)(\bar{\Phi}\gamma^5\Psi) \tag{A1}$$

The identity (A1) can be written as follows:

$$\bar{\chi}\{\gamma^\mu(\bar{\Phi}\gamma_\mu\Psi)\}\Psi=\bar{\chi}\{(\bar{\Phi}\Psi)+\gamma^5(\bar{\Phi}\gamma^5\Psi)\}\Psi \tag{A2}$$

and since it holds for any spinor χ, one has that for any two arbitrary spinors Φ and Ψ the following identity holds:

$$\{\gamma^\mu(\bar{\Phi}\gamma_\mu\Psi)\}\Psi = \{(\bar{\Phi}\Psi) + \gamma^5(\bar{\Phi}\gamma^5\Psi)\}\Psi \tag{A3}$$

By taking

$$\Phi = \Psi \tag{A4}$$

(A3) reads

$$\{\gamma^\mu(\bar{\Psi}\gamma_\mu\Psi)\} = \{(\bar{\Psi}\Psi) + \gamma^5(\bar{\Psi}\gamma^5\Psi)\}\Psi \tag{A5}$$

and for equations (14)-(16) and (21), (A5) gives

$$(\gamma^\mu\xi_\mu)\Psi = e^{\gamma^5\alpha}\Psi \tag{A6}$$

Similarly, by putting

$$\Phi = \gamma^5\Psi \tag{A7}$$

one has, after (22),

$$(\gamma^\mu\eta_\mu)\Psi = -\gamma^5 e^{\gamma^5\alpha}\Psi \tag{A8}$$

APPENDIX B

The Dirac conjugate of equation (62) reads

$$\bar{\Psi}_{,\mu}\gamma^\mu = -i\Psi\gamma^\mu \frac{e^{-\gamma^5\alpha}}{\rho}(j_\mu + \gamma^5 g_\mu) - i(m - in)\bar{\Psi} \tag{B1}$$

which, after multiplication on the right by Ψ, gives

$$\begin{aligned}\bar{\Psi}_{,\mu}\gamma_\mu\Psi = ij_\mu \frac{1}{\rho}(\bar{\Psi}\gamma^\mu\, e^{-\gamma^5\alpha}\Psi) - g_\mu \frac{1}{\rho}(\bar{\Psi}\gamma^\mu\, e^{-\gamma^5\alpha}\gamma^5\Psi)\\ - i(m - in)(\bar{\Psi}\Psi)\end{aligned} \tag{B2}$$

Similarly, by multiplying equation (62) on the left by Ψ, one has

$$\begin{aligned}\bar{\Psi}\gamma^\mu\Psi_{,\mu} = ij_\mu \frac{1}{\rho}(\bar{\Psi}\gamma^\mu\gamma^5\, e^{\gamma^5\alpha}\Psi) - ig_\mu \frac{1}{\rho}(\bar{\Psi}\gamma^\mu\gamma^5\, e^{\gamma^5\alpha}\Psi)\\ + i(m - in)(\bar{\Psi}\Psi)\end{aligned} \tag{B3}$$

and by adding (B2) and (B3), one has

$$\begin{aligned}\bar{\Psi}\gamma^\mu\Psi_{,\mu} = ij_\mu \frac{1}{\rho}\{\bar{\Psi}\gamma^\mu(e^{\gamma^5\alpha} - e^{-\gamma^5\alpha})\Psi\}\\ - ig_\mu \frac{1}{\rho}\{\bar{\Psi}\gamma^\mu(e^{\gamma^5\alpha})\gamma^5\Psi\} + 2n(\bar{\Psi}\Psi)\end{aligned} \tag{B4}$$

From equations (14)-(16) one also has

$$e^{\gamma^5\alpha} + e^{-\gamma^5\alpha} = 2\cos\alpha \tag{B5}$$

and

$$e^{\gamma^5\alpha} - e^{-\gamma^5\alpha} = 2\gamma^5 \sin\alpha \tag{B6}$$

so that equation (B4) reads

$$(\bar{\Psi}\gamma^\mu\Psi)_{,\mu} = -2i(j_\mu \sin\alpha - g_\mu \cos\alpha)\eta^\mu + 2n\rho\cos\alpha \tag{B7}$$

Similarly one has

$$(\bar{\Psi}\gamma^5\gamma^\mu\Psi)_{,\mu} = 2i(j_\mu \sin\alpha - g_\mu \cos\alpha)\xi^\mu + 2im\rho\sin\alpha \tag{B8}$$

For sake of simplicity, we take

$$A_\mu = \frac{1}{\rho}(j_\mu \sin\alpha - g_\mu \cos\alpha) \tag{B9}$$

so that equations (B7) and (B8) read

$$(\bar{\Psi}\gamma^\mu\Psi)_{,\mu} = -2i\rho A_\mu \eta^\mu + 2n\rho\cos\alpha \tag{B10}$$

and

$$(\bar{\Psi}\gamma^5\gamma^\mu\Psi)_{,\mu} = 2i\rho A_\mu \xi^\mu + 2im\rho\sin\alpha \tag{B11}$$

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